ON THE LOCAL HEAT BALANCE

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ABSTRACT

The heat absorbed by the air during the diurnal temperature rise is computed for Oklahoma City. Insolation measurements are recorded for each day, and the energy that is not used in raising the temperature is assumed to represent an empirical integration of the other terms of the local energy balance. With this empirical value taken to be the sum of energy losses, computations are made to show the ratio of the energy loss to the total insolation. These ratios of energy loss are correlated to the observed states of the ground. The computations are made for 141 days taken at random during one year. Eight ground states are used in the analysis and significance tests. The results show that the state of the ground affects the energy balance to such an extent that errors of 4° F. or more in maximum temperature forecasts may be due entirely to the effect of the state of the ground.

1. INTRODUCTION

Neiburger [1] calculated the part of the total solar energy received at the ground that is available to raise the air temperature (near the surface) to the afternoon maximum and to increase the lapse rate of the heated layer to dry adiabatic. He did this by computing and estimating average values for each term in the energy balance equation with the residue considered to be effective in heating the layer. The verification cases listed by Neiburger were restricted to cloudless days in the summer season when wind was light. In view of these nearly ideal conditions, good results might be expected, but, of course, Neiburger's purpose was to demonstrate what could be done with calculations of a local energy balance. It follows from Neiburger's energy balance and energy-area equations that similar results should be expected for widely varying conditions. The fact that less success is sometimes encountered under varying conditions suggests that more study is needed. Neiburger suggested that the energy absorbed by the heated layer should be established empirically and that the balance of the insolation could then be considered a physical integration of the other terms in the energy balance equation.

Myers [2, 3] calculated an empirical curve for the effective absorption of the heated layer for Nashville. He also discussed the variability of the other terms of the energy balance equation, but these variabilities did not affect the empirical curve which was a smoothed annual march of the effective absorption.

Williams [4] described a technique for application of heat-balance computations to short-range temperature forecasting and obtained an impressive verification on test forecasts. His computations were similar to Neiburger's, although he used more recent tables to obtain values of albedo and insolation. Kleinsasser and Younkin [5] have used Myers' approach to forecast maximum temperatures in the TVA power distribution area.

This present paper is a study of local heat balance computations for Oklahoma City and, in a sense, is a continuation of the work of Neiburger, Myers, and Williams. The objective here is to evaluate the effect of the state of ground on the energy loss that does not contribute to raising the air temperature. The procedure is to determine empirically the sum of the energy losses and to correlate the state of the ground with the ratio of energy loss to total insolation.

2. DATA AND COMPUTATIONS

The local energy balance at the ground is represented by the following equation:

$$I_0 = RI_0 + B_0 + LE + S + H_0 \tag{1}$$

where I_0 is the total incident solar radiation per unit surface, R is the surface albedo, B_0 is the net long-wave radiation (positive upward), E is the amount of water evaporated or transpired, L is the latent heat of evaporation, S is the heat used in raising the temperature of the ground, and H_0 is the heat available from the ground surface to raise the temperature of the lower atmosphere and to offset advective and radiative changes in the energy of the heated layer. It follows, then, that the energy ϕ not available to the air of the heated layer is

$$\phi = I_0 - H_0 = RI_0 + B_0 + LE + S \tag{2}$$

In this survey values of ϕ were determined from meas-

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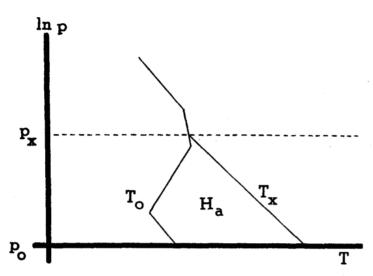


FIGURE 1.—Emagram of a typical case of heating H_a of layer of air from initial temperature T_0 to assumed afternoon maximum temperature T_x . After correction for advective and radiative effects in the layer, the energy H_a is taken as a measure of H_0 , the heat available from the surface to the heated layer

urements of I_0 and estimations of H_0 . The data for I_0 were taken from an Eppley pyrheliometer. H_0 was estimated from the temperature change on an emagram as illustrated in figure 1.

The quantity of heat H_a used in raising the temperature of the layer is given by:

$$H_{a} = \frac{c_{p}}{g} (p_{0} - p_{x}) (\overline{T}_{x} - \overline{T}_{0})$$

$$(3)$$

where c_p is the specific heat at constant pressure, g is the acceleration due to gravity, p_0 is the surface pressure, p_x is the pressure at the top of the layer affected by surface heating, \overline{T}_0 is the initial mean temperature of the layer, and \overline{T}_x is the maximum mean temperature of the layer. Assuming $c_p = .2417$ cal. gm.⁻¹(° C.)⁻¹ for moist air, g = .980 cm. sec.⁻², and converting pressure units to millibars and temperature units to degrees Fahrenheit:

$$H_a = .1372(p_0 - p_x)(\overline{T}_x - \overline{T}_0) \tag{4}$$

where H_a is in langleys.

Figure 1 presents an emagram of a typical case in which heat H_0 from the surface increases the lapse rate until it is presumably near dry-adiabatic at the time of the afternoon maximum temperature. The area between the curves is proportional to the change in energy H_a . Therefore, the difference in the means of the temperature curves, T_0 and T_z , may be taken as a measure of the temperature change $(\overline{T}_z - \overline{T}_0)$, to be used in equation (4). In general however, $H_a \neq H_0$ because of advective temperature changes and short-wave and long-wave radiation effects on the energy balance of the layer. (See for example Myers' [3] equation (2).) Therefore corrections, to be explained later, were applied to T_0 to offset these effects. With these corrections taken into account, equation (4) yields $H'_a \approx H_0$.

Data for 141 days were tabulated during a 1-year period beginning February 11, 1953, and ending February 10, 1954. The data were recorded from the observation forms of the Weather Bureau Airport Station, Oklahoma City, each time the writer verified the forms as a routine office duty. Since verification duty was rotated, the selection of data for the survey was assumed to be random, except for a leave period during July and August in which no data were recorded.

The selected data were believed to be unbiased because weather conditions were not a determining factor in the selection. Daily cloudiness varied from clear to overcast. Variations in cloudiness were not believed to influence the correlation significantly, first, because the selection of days was random, and second, because cloudiness subtracts from the insolation total, I_0 and its effect is therefore reflected in the pyrheliometer measurement.

The data for the initial temperature curve, T_0 , were taken from the 1500 gmr radiosonde observations to which advection corrections were applied. The amount of the correction was arbitrarily taken to be one-half of the difference between the mean temperature of the layer at 1500 gmr and the mean temperature of the same layer 12 hours later. This difference, however, is also affected by short-wave and long-wave radiation, so it was necessary to make adjustments to the mean 0300 gmr temperatures based on the diurnal temperature variation of the layer. For these radiation adjustments to the advection corrections, monthly means of the differences between the mean-layer temperatures at 1500 and 0300 gmr were computed. The monthly mean differences were between 1° F. and 4° F.

The maximum-temperature curve, T_x , for each case was assumed to be the potential temperature line corresponding to the observed surface afternoon maximum temperature. These curves were not the precise limits of the diurnal energy areas, but they were sufficiently representative to show the correlation required.

From the values of I_0 and H_0 , ϕ was determined from equation (2) and the ratio $100\phi/I_0$ was computed. This ratio was used as the dependent variable in the statistical analysis.

The data for the independent variable, the state of the ground, were taken from the observation forms (WBAN 10B, Col. 50, 1200 GMT) at the Weather Bureau Airport Station, Oklahoma City. The observations were adjusted in cases where the state of the ground changed abruptly soon after the observation. The classifications dry ground, wet ground, and puddles are identical to codes 0, 1, and 2 of the international synoptic code (Weather Bureau Circular N, table 11-2). These classifications contain 91 percent of the total data. The remaining data were cases in which fusion may have been a factor. Because there were only 13 of these cases, they were classified into only two categories: Surface of snow or slush, and frozen ground or heavy frost. For the same reason, conclusions as to the effects of fusion are possibly less reliable than other results.

Table 1.—Relation between the ratio 100 ϕ/I_0 and the state of the ground as shown by frequency of contingent categories of the variables

Bars show the mean of each column.

					ren Se	ason		Grow	ing Se	ason	cy
100 Ø 1 ₀ (%)	đ	Sfc. of	snow or slush	Frzn gnd or hvy frost	Puddles	Wet gnd	Dry gnd	Puddles	Wet gnd	Dry gnd	Class frequency
96 to 100	5	шш	3							-	3
91 to 95	4								·		1.11.1
86 to 90	3	шш	1	1		1	3	1	1	1	9
81 to 85	2	шШ		2	1		4		4	8	3 9 20 37 43 13 11 2 1
76 to 80	1		1	2	3_	1	6	2	7	15	37
71 to 75	0			3	2	8	7	·	5	_18_	43
66 to 70	- 1						5		1	7	13
61 to 65	-2					1	5			5	11
56 to 60	-3	Ē_					1			1	2
51 to 55	-4	<u>E</u> _					1				1
4 6 to 50	-5	E_			i		1				l =
41 to 4 5	-6	E									11111
3 6 to 4 0	-7									l	1
Col free	1		5	8	6	11	33	4	18	56	141
Mean (%)	9	3	79	77	74	72	83	78	73	75
Variance	,	6	4	19	8	47	92	17	23	62	79
Sum of square	8	32	20	222	71	341	3088	71	424	3503	10 26 9

Separate tabulations were used for ground states during the barren season and the growing season which began April 4, 1953, and ended November 8, 1953. Although a freeze occurred at Oklahoma City on April 18, most plants, especially the pastures, turned green again almost immediately and the growing season was virtually uninterrupted.

The tabulation of the data according to the value of the ratio $100\phi/I_0$ and the state of the ground is given in table 1. The results were analyzed statistically by the technique of analysis of variance and the significance determined by the F-ratio and appropriate tables (c. f. Panofsky and Brier [6]). The results of this analysis are

given in table 2. The F-ratio of 5.3 exceeds the F-table value of 2.8 at the 1 percent level of significance.

Seasonal variations of insolation and variation in time

Table 2.—Analysis of variance of the data in table 1.

Source of variance	Degrees of freedom	Sum of squares	Mean square	F-ratio
Between columns Within columns	7 133	2229 8040	319 60	5. 3
Totals	140	10269		

Note: F-table_value_at 1_percent level_of significance=2.8.

Table 3.—Seasonal variation of insolation and absorption (after Myers [2]). Insolation and heat values are in langleys.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Insolation (Daily totals) (I ₀) Insolation (Sunrise to 4 p. m., T. S. T.)		369		614		716		647		395		251
(I')		360		574		652		597		383		248
Heat from surface absorbed by lower atmos. (H_0)		108		190		212		215		115		50
Sum of heat losses $(\phi' = I_0' - H_0)$		252		384		440	 	382		268		198
$\frac{100 \ \phi'}{I_0'}$, Sunrise to 4 p. m., (T. S. T.)		70	69	67	67	67	66	64	67	70	75	80
Deviation from mean Seasonal correction	+5 -5	0	-1 +1	$^{-3}_{+3}$	-3 +3	-3 +3	-4 +4	-6 +6	-3 +3	0	+5 -5	+10 -10

of sunset also affected the ratio $100 \phi/I_0$. Myers' [2] curves for the annual march of daily insolation and daily heating (table 3) were used to evaluate these effects on the ratio. Monthly deviations from the annual mean value of 70 percent were added to each value of the ratio for the 141 cases, and the variance-ratio was recomputed (tables 4 and 5) in the same manner as in tables 1 and 2. Myers' data were assumed to be applicable at Oklahoma City because Oklahoma City and Nashville are very near the same latitude. Some error may be introduced by this assumption because cloudiness is a factor and the seasonal variation in cloudiness is not identical at the two stations.

Means were not computed in table 4 because the data were not completely empirical. The means of table 1 resulted directly from measurements and were, therefore, the empirical results sought. The increase in the variance-ratio (F) as shown by table 5 suggests that it would be desirable to compute means for the ratio $100 \phi/I_0$ for each month, but a great deal more data would be needed. The increase in the variance-ratio (F) in table 5 indicates

Table 4.—Relation between the ratio 100 ϕ/I_0 and the state of the ground after applying seasonal corrections as shown by frequency of contingent categories.

			Bar	ren seaso	Grov					
100 ϕ/I 0 (percent)	d	Sfc of snw or slush	Frzn gnd or hvy frost	Puddles	Wet gnd	Dry gnd	Puddles	Wet gnd	Dry gnd	Class freq.
91/95	4	3					1	1	1	6
86/90	3						1	2	3	6
81/85	2	1	1	1	1	3	1	4	10	22
76/80	1	1	4	3	3	4	1	7	18	41
71/75	0		2	2	4	11		4	13	36
66/70	-1				2	7			6	15
61/65	-2		1		1	2			4	8
56/60	-3					2				2
51/55	-4					2				2
46/50	-5					2				2
41/45	-6									
36/40	-7								1	1
Col. freq		5	8	6	11	33	4	18	56	141
Sum of sq	rs	200	250	71	323	2,802	125	557	3, 898	11, 127

Table 5.—Analysis of variance of the data in table 4

Source of variance	Degrees of freedom	Sum of squares	Mean square	F-ratio
Between columns	7	2, 901	414	6.7
Within columns	133	8, 226	62	
Totals	140	11, 127		

Note: F-table value at 1 percent level of significance = 2.8

further that the decrease in the effectiveness of insolation in raising temperature during the growing season is even more pronounced than indicated in table 1.

Table 6 is a comparison of data used in this paper with data used by Myers [2] and Williams [4]. The data shown were average values for the month of June with adjustments made so that values of insolation and absorption between sunrise and 4:00 p.m. (True Solar Time) were comparable. Corrections for latitude were not necessary since the three stations, Nashville, Oklahoma City, and Las Vegas, are all near the 36th parallel. No corrections were made for the differences in longitude even though the differences in solar time of radiosonde observations may have been a factor in the comparisons. Values for H_0 were affected by local differences in terrain, soil, and vegetation. Insolation reaching the ground was greatest at Las Vegas—a result which conforms to altitude and humidity considerations. In addition to this a greater part of the insolation was used to increase air temperature at Las Vegas. Williams makes note of this fact in describing the extremely dry, sandy soil conditions. Hembree's averages for I_0 and H_0 were smallest as one may expect since data for days with cloudiness were included.

Since the data used in this paper for I_0 and H_0 have not been included in the tables, a brief summary is included here. The mean of the insolation measurements, I_0 , was 431 langleys; the monthly means ranged from 256 to 615; the absolute range was from 121 to 701. The mean of the computed values of H_0 was 107 ly.; the monthly means ranged from 49 to 151; the absolute range was from 2 to 273.

3. DISCUSSION

According to Neiburger [1], we may assume that inso-

lation is reflected, reradiated, absorbed by the ground, or lost in evaporation, and that the balance is effective in changing air temperature. The quantity ϕ was computed as an empirical integration of the energy loss terms and its variations appear to be the result of variations in surface albedo, evaporation losses, and surface conduction.

The within-column variance (table 2) of the data in table 1 was due partly to the effect of the long-wave radiation, partly to the arbitrary limits chosen for the diurnal energy area, and partly to the arbitrary manner in which advection corrections were made, and to variations in static stability and relative humidity.

From the statistical analysis, the F-test indicates that the correlation between $100 \phi/I_0$ and state of the ground is significant. The F-ratio of 5.3 in table 2 is nearly twice the F-table value of 2.8 at the 1 percent level of significance. The correlation can be applied in short-range forecasting to estimate the effect of insolation in temperature forecasting.

Objective methods for forecasting temperatures by the use of radiation data have been discussed in detail by Myers [3], Williams [4], and Kleinsasser and Younkin [5]. As with any objective technique, accuracy is dependent entirely on the validity of the assumptions and the representativeness of the data used. If the assumptions are accurate and if all the data are representative to within fractions of units, objective temperature forecasts that are accurate to within a fraction of a degree should be routine.

Often empirical data uncover errors in assumptions. For example, there is an equation used by Williams [4], and by many other writers, that assumes a linear relationship between amount of cloud cover and the albedo of the clouds. Measurements show that the relationship is non-linear. Kimball [7] gives the following measurements of insolation at Washington for days without cloudiness ranging from 0 to 10 tenths:

These figures show that the assumption of linearity may cause appreciable error. Scattered cloudiness has but slight effect on insolation, broken cloudiness permits most of the insolation to penetrate, yet an overcast reflects a large fraction of the energy (c. f. Fritz [8])—a fact well known to temperature forecasters.

The ellipse,

$$y=1.18\sqrt{1-\frac{(x+0.3)^2}{(1.4)^2}}-0.15,$$

where y is the fraction of clear-day insolation received and x is the fraction of the sky covered with clouds, is a good fit for Kimball's results. This equation may be used to estimate the effect of cloud cover on insolation with better results than the more popular linear equation, but the eccentricity may vary with locality. Fritz [9] discussed

Table 6.—A comparison of the June data of three authors. (Period: Sunrise to 4:00 p. m., true solar time)

	Unit	Williams [4]	Myers [2]	Hembree
Insolation (I' ₀)	ly.	759	652	503
Heat from surface absorbed by lower atmosphere (H_0) . Sum of heat losses (ϕ') .	ly. ly.	512 247	212 440	128 375
$\frac{100 \phi'}{I_0'}$		33	67	75

the nonlinearity of this relation, but he found that the relation between I_0 and the duration of sunshine was linear. He suggested that the difference may lie in the fact that a sunshine recorder is sensitive to insolation passing through thin clouds. Accordingly, the curvature of a statistical relation between I_0 and sky cover may be a function of the station's cloud regime. At the present time there are more that 5 years of data punched on the "solar cards" from the stations equipped with pyrheliometers. Each card contains the average daily cloud cover, duration of sunshine, and the data necessary to compute the fraction of clear-day insolation received. A group of empirical curves, showing any desired degree of detail, can be computed for any of the stations in about 8 minutes time by the computers currently being used for this type of problem.

A simple solution to the temperature forecasting problem by statistical analysis is not possible due to the number of variables and lack of observations. Therefore, the quantity ϕ , the sum of energy losses, is a convenient parameter, although it is not entirely satisfactory due to the interrelation of some of the variables.

Table 1 gives the relationship between the ratio $100 \phi/I_0$ and the state of the ground as a column mean regression, and the regression may be used to estimate the effect of the state of the ground in forecasting an element, such as maximum temperature, for which the effectiveness of insolation is an important argument.

Consider, for example, a simple maximum temperature forecast in which the effectiveness of the insolation in warming the air is the only argument. Suppose that the ground is dry and that a temperature rise of 20° F. is forecast (a change of 10° F. in the mean of the heated layer). T_0 is known. Therefore, the forecasting of T_x and the lapse rate of T_x (which is usually dry-adiabatic) is equivalent to forecasting the pressure interval and the area H_0 . It follows, then, from equation (4) and table 1 that 612 ly. of insolation are forecast and that 27 percent of the incident energy will be used in warming the air. But, if a hard shower during the night has left puddles of water on the ground, an effectiveness of 17 percent may be expected, or a temperature rise of only 16° .

Accordingly, the state of the ground should be considered as an independent variable in the formulation of any scheme for forecasting an element, such as maximum

temperature, that depends on the effectiveness of insolation. This is especially true if very small error is desired. However, attention would have to be given to the large within-column variance of table 2 and to the sources of this variance. The most obvious sources of the variance are listed above.

To suggest merely that additional study is needed is to greatly deemphasize the problem of forecasting temperatures. It is extremely desirable to forecast temperatures with errors less than 1°. To do this, however, it is necessary to formulate the interrelationships of the variables as they occur in the available measurements of observed conditions. The regression in table 1 between the state of the ground and the effectiveness of the insolation in heating the air is a portion of this larger problem and demonstrates how a single variable, the state of the ground, acts independently on three terms, RI_0 , LE, and S, to disturb the energy balance.

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